ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
(ISRA), Impact Factor: 2.114

IJESRT

INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

BRANCH AND BOUND TECHNIQUE FOR SINGLE MACHINE SCHEDULING PROBLEM USING TYPE-2 TRAPEZOIDAL FUZZY NUMBERS

R. Helen*, R.Sumathi

* PG and Research Department of Mathematics, Poompuhar College (Autonomous), Melaiyur, India
P.G. Department of Mathematics, R.V.S.College of Engg. and Technology, Karaikal

ABSTRACT

This paper deals branch and bound technique to solve single machine scheduling problem involving two processing times along with due date using Type-2 Trapezoidal fuzzy numbers. Our aim is to obtained optimal sequence of jobs and to minimize the total tardiness. The working of the algorithm has been illustrated by numerical example.

AMS Subject Classification: 94D05, 90-XX, Aug 2010.

KEYWORDS: Branch and Bound Technique, Processing times P1 and P2 , Optimal sequence, Type-2 Trapezoidal fuzzy numbers.

INTRODUCTION

Job scheduling is a useful tool in decision making problem. The scheduling problems are common occurrence in our daily life. The aim of this technique is used to determine an optimal job scheduling problem and minimizing the total tardiness. For many years, Scheduling problem focused on single performance measure.

In this paper, we propose a new concept in single machine scheduling problem. Recent development of new technology, we are consider the single machine having double processor to do two different works to complete a job. Each work having separate processing times (ie) P1 and P2 addition to the due date (dj). The most obvious objective is to scheduling the job and minimizing the total tardiness using Branch and Bound technique. This method is basically a stage wise search method of optimization problems whose solutions may be viewed as the result of a sequence of decisions that will help the decision maker in determining a best schedule for a given set of jobs effectively. This method is become lucrative to make decision. In most of the real life problem, there are elements of uncertainty in process. In practical situation processing times and due date are not always deterministic. So, we have associated with fuzzy environment.

The concept of a type-2 fuzzy set, which is an extension of the concept of an ordinary fuzzy set, was introduced by Zadeh [16]. A fuzzy set is two dimensional and a type-2 fuzzy set is three dimensional, type-2 fuzzy sets can better improve certain kinds of inference than do fuzzy sets with increasing imprecision, uncertainty and fuzziness in information. A type-2 fuzzy set is characterized by a membership function, (ie) the membership value for each element of this set is a fuzzy set in [0.1], unlike an ordinary fuzzy set where the membership value is a crisp number in [0,1].

REVIEW OF LITERATURE

Various researchers have done a lot of work in different directions. Ishii and Tada [6] considered a single machine scheduling problem minimizing the maximum lateness of jobs with fuzzy precedence relations. Hong et.al., [5] introduced a single machine scheduling problem with fuzzy due date. Itoh and Ishii [7] proposed a single machine scheduling problem dealing with fuzzy processing times and due date. Gawiejnowicz et.al., [3] deals with a single machine time dependent scheduling problem. Emmons [2] developed several theorems and dominance rules that can be used to restrict the search effort of a branch and bound algorithm. Lawler [10] applied a dynamic programming approach to the single machine total tardiness problem. Vaiaraktarakis and Chung [15] proposed a branch and bound algorithm to minimize total tardiness subject to minimum number of tardy jobs. Azizogulu [1] used a branch and

ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
(ISRA), Impact Factor: 2.114

bound method to solve the total earliness and total tardiness problem for the single machine problem. Raymond [14] proposed a branch and bound approach to solve the problem for steel plant involving single machine bi-criteria problem.

The paper is organized as follows: In section 2, deals with the preliminaries. In section 3, arithmetic operations on type-2 trapezoidal fuzzy number and ranking function are discussed. In section 4, we introduced a brief note on Branch and Bound Technique. In section 5, the effectiveness of the proposed method is illustrated by means of an example.

PRELIMINARIES

Definition: Fuzzy Set

A fuzzy set is characterized by a membership function mapping the elements of domain, space or universe of discourse X to the unit interval [0,1].

A fuzzy set $\tilde{A}$ is set of ordered pairs { (x, $\mu_{\tilde{A}}$(x)) / x ε R} where $\mu_{\tilde{A}}(x): R \to [0,1]$ is upper semi continuous function, $\mu_{\tilde{A}}(x)$ is called a membership function of the fuzzy set.

Definition: Fuzzy Number

A fuzzy number f in the real line R is a fuzzy set f: R→[0,1] that satisfies the following properties.

(i) f is piecewise continuous.
(ii) There exists an x ε R such that f(x) =1.
(iii) f is convex (i.e) if x_1 , x_2 ε R and then λ ε [0,1] then
$f(\lambda x_1 + (1-\lambda) x_2) \geq f(x_1) \wedge f(x_2)$.

Definition: Type-2 Fuzzy Set

The type-2 fuzzy sets are defined by functions of the form $\mu_A : X \to ƛ([0,1])$ where $ƛ([0,1])$ denotes the set of all ordinary fuzzy sets that can be defined within the universal set [0,1]. An example of a membership function of this type is given in fig-1.

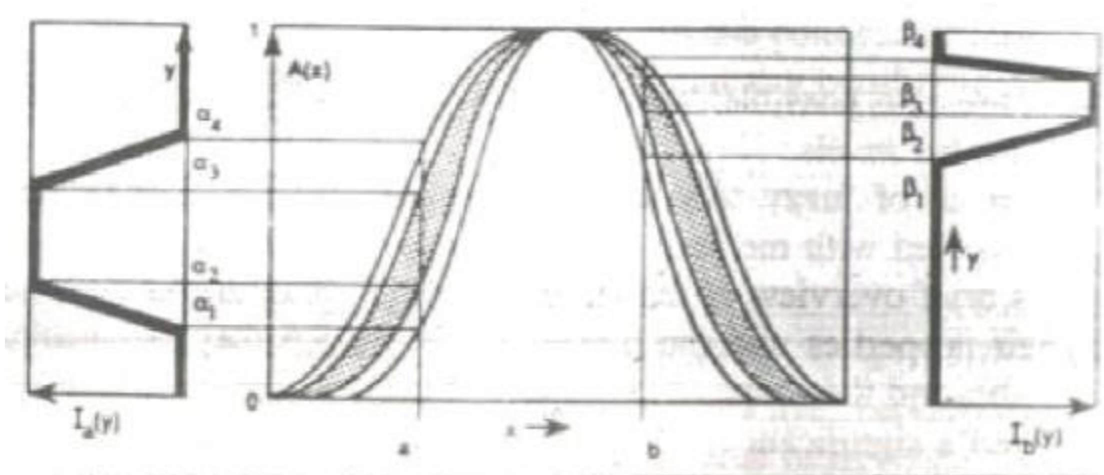

Illustration of the concept of a fuzzy set of type 2.

Definition: Type-2 Fuzzy Number

Let $\tilde{\tilde{A}}$ be a type-2 fuzzy set defined in the universe of discourse R. If the following conditions are satisfied.

(i) $\tilde{\tilde{A}}$ is normal.

ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
(ISRA), Impact Factor: 2.114

(ii) $\tilde{\tilde{A}}$ is a convex set.

(iii) The support of $\tilde{\tilde{A}}$ is closed and bounded, then $\tilde{\tilde{A}}$ is called a type-2 fuzzy number.

Definition: Trapezoidal Fuzzy Number

A trapezoidal fuzzy number A = (a_1,a_2, a_3,a_4) whose membership function is given by

$$\mu_A(x) = \begin{cases} 0 & , x < a_1 \ \ \& \ \ x > a_4 \\ \dfrac{x-a_1}{a_2-a_1} & , a_1 \le x \le a_2 \\ 1 , & a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3} & , a_3 \le x \le a_4 \end{cases}$$

Definition: Type-2 Trapezoidal Fuzzy Number

A type-2 trapezoidal fuzzy number $\tilde{\tilde{A}}$ on R is given by

$$\tilde{\tilde{A}} = \{ x, \mu_{\tilde{A}_1}(x), \mu_{\tilde{A}_2}(x), \mu_{\tilde{A}_3}(x), \mu_{\tilde{A}_4}(x)), \quad x \, \varepsilon R \quad \}$$ and

$\mu_{\tilde{A}_1}(x) \le \mu_{\tilde{A}_2}(x) \le \mu_{\tilde{A}_3}(x) \le \mu_{\tilde{A}_4}(x)$ for all x εR. (ie) $\tilde{\tilde{A}} = \left(\tilde{A}_1, \tilde{A}_2, \tilde{A}_3, \tilde{A}_4\right)$ where

$\tilde{\tilde{A}}$ = ((a_1^L, a_1^M , a_1^N ,a_1^U), (a_2^L ,a_2^M ,a_2^N ,a_2^U), (a_3^L , a_3^M ,a_3^N ,a_3^U), (a_4^L ,a_4^M, a_4^N,a_4^U)).

ARITHMETIC OPERATIONS

Arithmetic Operations on Type-2 Trapezoidal Fuzzy Numbers:

Let $\tilde{\tilde{A}} = (\tilde{A}_1, \tilde{A}_2, \tilde{A}_3, \tilde{A}_4)$

= ((a_1^L,a_1^M,a_1^N,a_1^U),(a_2^L,a_2^M,a_2^N,a_2^U),(a_3^L,a_3^M,a_3^N,a_3^U),(a_4^L,a_4^M,a_4^N,a_4^U))

& $\tilde{\tilde{B}} = \left(\tilde{B}_1, \tilde{B}_2, \tilde{B}_3, \tilde{B}_4\right)$

= ((b_1^L, b_1^M, b_1^N,b_1^U),(b_2^L,b_2^M,b_2^N, b_2^U),(b_3^L,b_3^M,b_3^N,b_3^U),(b_4^L,b_4^M,b_4^N,b_4^U))

be two type-2 trapezoidal fuzzy numbers. Then, we define

Addition:

$\tilde{\tilde{A}} + \tilde{\tilde{B}}$ = {($a_1^L + b_1^L$, $a_1^M + b_1^M$, $a_1^N + b_1^N$, $a_1^U + b_1^U$), ($a_2^L + b_2^L$, $a_2^M + b_2^M$, $a_2^N + b_2^N$, $a_2^U + b_2^U$), ($a_3^L + b_3^L$, $a_3^M + b_3^M$, $a_3^N + b_3^N$,$a_3^U + b_3^U$), ($a_4^L + b_4^L$, $a_4^M + b_4^M$, $a_4^N + b_4^N$, $a_4^U + b_4^U$)}.

Subtraction:

$\tilde{\tilde{A}} - \tilde{\tilde{B}}$ = {($a_1^L - b_1^U$, $a_1^M - b_1^N$, $a_1^N - b_1^M$, $a_1^U - b_1^L$), ($a_2^L - b_2^U$, $a_2^M - b_2^N$, $a_2^N - b_2^M$, $a_2^U - b_2^L$),($a_3^L - b_3^U$, $a_3^M - b_3^N$, $a_3^N - b_3^M$, $a_3^U - b_3^L$), ($a_4^L - b_4^U$, $a_4^M - b_4^N$, $a_4^N - b_4^M$, $a_4^U - b_4^L$)}.

Multiplication:

$\tilde{\tilde{A}} x \tilde{\tilde{B}}$ = {($a_1^L * b_1^L$, $a_1^M * b_1^M$, $a_1^N * b_1^N$, $a_1^U * b_1^U$),($a_2^L * b_2^L$, $a_2^M * b_2^M$, $a_2^N * b_2^N$, $a_2^U * b_2^U$),($a_3^L * b_3^L$, $a_3^M * b_3^M$, $a_3^N * b_3^N$, $a_3^U * b_3^U$). ($a_4^L * b_4^L$, $a_4^M * b_4^M$, $a_4^N * b_4^N$, $a_4^U * b_4^U$)}.

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ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
(ISRA), Impact Factor: 2.114

$$\frac{\approx\atop{A}}{\approx\atop{B}} = \left\{\left(\frac{a_1^L}{b_1^U},\frac{a_1^M}{b_1^N},\frac{a_1^N}{b_1^M},\frac{a_1^U}{b_1^L}\right),\left(\frac{a_2^L}{b_2^U},\frac{a_2^M}{b_2^N},\frac{a_2^N}{b_2^M},\frac{a_2^U}{b_2^L}\right),\left(\frac{a_3^L}{b_3^U},\frac{a_3^M}{b_3^N},\frac{a_3^N}{b_3^M},\frac{a_3^U}{b_3^L}\right),\left(\frac{a_4^L}{b_4^U},\frac{a_4^M}{b_4^N},\frac{a_4^N}{b_4^M},\frac{a_4^U}{b_4^L}\right)\right\}.$$

Ranking on Type-2 Trapezoidal Fuzzy Number

Let F(R) be the set of all type-2 normal trapezoidal fuzzy numbers. One convenient approach for solving numerical valued problem is based on the concept of comparison of fuzzy numbers by use of ranking function. An effective approach for ordering the elements of F(R) is to define a linear ranking function R: F (R) → R which maps each fuzzy number in to R.

Suppose if $\tilde{\tilde{A}} = \left(\tilde{A}_1, \tilde{A}_2, \tilde{A}_3, \tilde{A}_4\right)$

$= ((a_1^L, a_1^M, a_1^N, a_1^U), (a_2^L, a_2^M, a_2^N, a_2^U), (a_3^L, a_3^M, a_3^N, a_3^U), (a_4^L, a_4^M, a_4^N, a_4^U))$.

Then we define

$$R(\tilde{\tilde{A}}) = a_1^L + a_1^M + a_1^N + a_1^U + a_2^L + a_2^M + a_2^N + a_2^U + a_3^L + a_3^M + a_3^N + a_3^U + a_4^L + a_4^M + a_4^N + a_4^U / 16.$$

Also, we define orders on F (R) by

$R(\tilde{\tilde{A}}) \geq R(\tilde{\tilde{B}})$ if and only if $\tilde{\tilde{A}} \geq \tilde{\tilde{B}}$

$R(\tilde{\tilde{A}}) \leq R(\tilde{\tilde{B}})$ if and only if $\tilde{\tilde{A}} \leq \tilde{\tilde{B}}$

$R(\tilde{\tilde{A}}) = R(\tilde{\tilde{B}})$ if and only if $\tilde{\tilde{A}} = \tilde{\tilde{B}}$

BRANCH AND BOUND TECHNIQUE

Branch and Bound: Branching is the process of partitioning a large problem into two or more subproblems and Bounding is the process of calculating a lower bound on the optimal solution of a given subproblems.

Dominance Property: While subdividing a subproblem P_σ^k into (n-k) subproblems, a careful analysis would help us to create only one subproblem instead of n-k subproblems. This is called dominance property. This will reduce the computational effort to a greater extent. In a subproblem P_σ^k, if there exists a job $i \,\varepsilon\, \sigma^1$ such that $d_i \geq q_\sigma$, then it is sufficient to create only one subproblem $P_{i\sigma}^{k+1}$. The remaining subproblems under P_σ^k can be ignored. In the bounding process, $V_{i\sigma} = V_\sigma$.

Tardiness: Tardiness is the lateness of job j if it fails to meet its due date; otherwise, it is zero. It is defined as :

T_j = max { o,$c_j - d_j$ } = max { 0,L_j } which means

$$T_j = \begin{cases} c_j - d_j & ,if \;\; c_j > d_j \\ 0 & ,otherwise \end{cases}$$

Notations:

n	:	The total number of independent jobs.
j	:	Represents the j^{th} job, j =1,2,….N.
t_j	:	The Processing time of the job j.
d_j	:	The due date of the job j.
C_j	:	The Completion time of the job j
T_j	:	The Tardiness of the job j
NT_j	:	Number of the tardy jobs.
$T_{m,n}$	:	Minimum tardiness.

ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
(ISRA), Impact Factor: 2.114

J	:	Location of i^{th} job on machine k.
N	:	Total number of jobs to be scheduled.
K	:	Machine on which i^{th} job is assigned at position j.
σ	:	The set of Scheduled jobs at the end of the sequence.
σ'	:	The set of unscheduled jobs. (or) complement of σ .
$q_\sigma = q_\varphi$	:	The sum of the processing times of unscheduled jobs in σ'.
$i\sigma$	:	Partial sequence in which σ is immediately preceded by job i.
$S(\sigma)$	:	The sum of the tardiness values of the jobs
P_σ^K	:	A subproblem at level k in the branching tree. In this subproblem the last k positions in the sequence are assigned some jobs.
V_σ	:	Value associated with p_σ^k, which is combination of jobs to total tardiness.

Algorithm

The processing times of jobs and due date are uncertain. This leads to the use of Type-2 trapezoidal fuzzy numbers for representing these imprecise values.

Step-1:

Place P_φ^0 on the active list; its associated values are: $V_\varphi = 0$ and $q_\varphi = \sum_{j=1}^{n} t_j$. At a given stage of the algorithm, the active list consists of all the terminal nodes of the partial tree created up to that stage.

Step-2 ;

Remove the first subproblem P_σ^k from the active list. If k is equal to n-1, stop. Prefix the missing job with σ and treat it as the optimal sequence. Otherwise, check the dominance property for P_σ^k . If the property holds, go to step 3; otherwise go to step 4.

Step -3:

Let the job j be the job with the largest due date in σ^1. Create the subproblem $P_{j\sigma}^{k+1}$ with $q_{j\sigma} = q_\sigma - t_j$, $V_{j\sigma} = V_\sigma^1$, $bj_\sigma = V\sigma$. Place $P_{j\sigma}^{k+1}$ on the active list, ranked by its lower bound. Return to step 2.

Step-4 :

Create (n-k) subproblems, one for each i € σ^1. For $P_{i\sigma}^{k+1}$, let, $q_{j\sigma} = q_\sigma - t_j$,
$V_{i\sigma} = V_\sigma + \max(0, q_\sigma - d_i)$, $b_{i\sigma} = V_{i\sigma}$. Now place each $P_{i\sigma}^{k+1}$ on the active list, ranked by its lower bound. Return to step -2.

NUMERICAL ILLUSTRATION

In milk producing factory, they required double processor for production using single machine. There are two processes done by the single machine. (i) Crushed the soya to produce milk is the first process made by the machine. (ii) That milk will be packed by respective quantities is the second process made by the same machine. These two processes are having separate processing times (P1, P2). Here, we consider the two processing time and due date with type-2 trapezoidal fuzzy numbers for each jobs are given in the following table:

Table-1:

Job j	Processing time P_1	Processing time P_2	Due date d_j
1	(5,6,10,11) (4,6,10,12) (3,6,10,13) (2,6,10,14)	(4,6,8,10) (3,6,8,11) (2,6,8,12) (1,6,8,13)	(14,18,22,26) (13,18,22,27) (12,18,22,28) (11,18,22,29)
2	(1,3,5,7) (0,3,5,8) (-1,3,5,9) (-2,3,5,10)	(1,3,5,7) (0,3,5,8) (-1,3,5,9) (-2,3,5,10)	(12,14,16,18) (11,14,16,19) (10,14,16,20) (9,14,16,21)
3	(5,8,10,13)	(4,7,9,12)	(12,24,36,48)

ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
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	(4,8,10,14) (3,8,10,15) (2,8,10,16)	(3,7,9,13) (2,7,9,14) (1,7,9,15)	(11,24,36,49) (10,24,36,50) (9,24,36,51)
4	(2,4,6,8) (1,4,6,9) (0,4,6,10) (-1,4,6,11)	(1,3,5,7) (0,3,5,8) (-1,3,5,9) (-2,3,5,10)	(5,13,21,29) (4,13,21,30) (3,13,21,31) (2,13,21,32)
5	(2,5,7,10) (1,5,7,11) (0,5,7,12) (-1,5,7,13)	(2,5,7,10) (1,5,7,11) (0,5,7,12) (-1,5,7,13)	(10,20,30,40) (9,20,30,41) (8,20,30,42) (7,20,30,43)

TABLE-2:

Job j	Processing time P_1	Processing time P_2	Processing time t_j	Due date d_j
1	(5,6,10,11) (4,6,10,12) (3,6,10,13) (2,6,10,14)	(4,6,8,10) (3,6,8,11) (2,6,8,12) (1,6,8,13)	(9, 12,18,21) (7,12,18,23) (5,12,18,25) (3,12,18,27)	(14,18,22,26) (13,18,22,27) (12,18,22,28) (11,18,22,29)
2	(1,3,5,7) (0,3,5,8) (-1,3,5,9) (-2,3,5,10)	(1,3,5,7) (0,3,5,8) (-1,3,5,9) (-2,3,5,10)	(2,6,10,14) (0,6,10,16) (-2,6,10,18) (-4,6,10,20)	(12,14,16,18) (11,14,16,19) (10,14,16,20) (9,14,16,21)
3	(5,8,10,13) (4,8,10,14) (3,8,10,15) (2,8,10,16)	(4,7,9,12) (3,7,9,13) (2,7,9,14) (1,7,9,15)	(9,15,19,25) (7,15,19,27) (5,15,19,29) (3,15,19,31)	(12,24,36,48) (11,24,36,49) (10,24,36,50) (9,24,36,51)
4	(2,4,6,8) (1,4,6,9) (0,4,6,10) (-1,4,6,11	(1,3,5,7) (0,3,5,8) (-1,3,5,9) (-2,3,5,10)	(3,7,11,15) (1,7,11,17) (-1,7,11,19) (-3,7,11,21)	(5,13,21,29) (4,13,21,30) (3,13,21,31) (2,13,21,32)
5	(2,5,7,10) (1,5,7,11) (0,5,7,12) (-1,5,7,13)	(2,5,7,10) (1,5,7,11) (0,5,7,12) (-1,5,7,13)	(4,10,14,20) (2,10,14,22) (0,10,14,24) (-2,10,14,26)	(10,20,30,40) (9,20,30,41) (8,20,30,42) (7,20,30,43)

Step-1:

Active list at level 0 = $\{P_\varphi^0\}$, $\sigma = \{\varphi\}$, $\sigma^1 = \{1,2,3,4,5\}$, $V_\varphi = 0$ & $q_\varphi = \begin{pmatrix} 27, 50, 72, 95 \\ 17, 50, 72, 105 \\ 7, 50, 72, 115 \\ -3, 50, 72, 125 \end{pmatrix}$.

Since, the current level k(0) is not equal to n-1(4). Check the dominance property. Also $\max_{i \in \sigma'} d_i = \begin{pmatrix} 12, 24, 36, 48 \\ 11, 24, 36, 49 \\ 10, 24, 36, 50 \\ 9, 24, 36, 51 \end{pmatrix}$. Since, this maximum is not greater than q_φ. The details of computations of the lower bound for each of the node is

ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
(ISRA), Impact Factor: 2.114

$Pi\sigma^1$	$V_{i\sigma} = V_\sigma + \max(0, q_\sigma - d_i)$	$b_{i\sigma} = v_{i\sigma}$
P_1^1	$0 + \max\left[0, \begin{pmatrix} 27,50,72,95 \\ 17,50,72,105 \\ 7,50,72,115 \\ -3,50,72,125 \end{pmatrix} - \begin{pmatrix} 14,18,22,26 \\ 13,18,22,27 \\ 12,18,22,28 \\ 11,18,22,29 \end{pmatrix}\right]$	$\begin{pmatrix} 1,28,54,81 \\ -10,28,54,92 \\ -21,28,54,103 \\ -32,28,54,114 \end{pmatrix}$
P_2^1	$0 + \max\left[0, \begin{pmatrix} 27,50,72,95 \\ 17,50,72,105 \\ 7,50,72,115 \\ -3,50,72,125 \end{pmatrix} - \begin{pmatrix} 12,14,16,18 \\ 11,14,16,19 \\ 10,14,16,20 \\ 9,14,16,21 \end{pmatrix}\right]$	$\begin{pmatrix} 9,34,58,83 \\ -2,34,58,94 \\ -13,34,58,105 \\ -24,34,58,116 \end{pmatrix}$
P_3^1	$0 + \max\left[0, \begin{pmatrix} 27,50,72,95 \\ 17,50,72,105 \\ 7,50,72,115 \\ -3,50,72,125 \end{pmatrix} - \begin{pmatrix} 12,24,36,48 \\ 11,24,36,49 \\ 10,24,36,50 \\ 9,24,36,51 \end{pmatrix}\right]$	$\begin{pmatrix} -21,14,48,83 \\ -32,14,48,94 \\ -43,14,48,105 \\ -54,14,48,116 \end{pmatrix}$
P_4^1	$0 + \max\left[0, \begin{pmatrix} 27,50,72,95 \\ 17,50,72,105 \\ 7,50,72,115 \\ -3,50,72,125 \end{pmatrix} - \begin{pmatrix} 5,13,21,29 \\ 4,13,21,30 \\ 3,13,21,31 \\ 2,13,21,32 \end{pmatrix}\right]$	$\begin{pmatrix} -2,29,59,90 \\ -13,29,59,101 \\ -24,29,59,112 \\ -35,29,59,123 \end{pmatrix}$
P_5^1	$0 + \max\left[0, \begin{pmatrix} 27,50,72,95 \\ 17,50,72,105 \\ 7,50,72,115 \\ -3,50,72,125 \end{pmatrix} - \begin{pmatrix} 10,20,30,40 \\ 9,20,30,41 \\ 8,20,30,42 \\ 7,20,30,43 \end{pmatrix}\right]$	$\begin{pmatrix} -13,20,52,85 \\ -24,20,52,96 \\ -35,20,52,107 \\ -46,20,52,118 \end{pmatrix}$

Active list = { $\{P_3^1, P_5^1, P_1^1, P_4^1, P_2^1\}$

Step-2:
Check the dominance property, $\sigma = \{3\}$, $\sigma^1 = \{1,2,4,5\}$,

$$q_\sigma = \begin{pmatrix} 27,50,72,95 \\ 17,50,72,105 \\ 7,50,72,115 \\ -3,50,72,125 \end{pmatrix} - \begin{pmatrix} 9,15,19,25 \\ 7,15,19,27 \\ 5,15,19,29 \\ 3,15,19,31 \end{pmatrix} = \begin{pmatrix} 2,31,57,86 \\ -10,31,57,98 \\ -22,31,57,110 \\ -34,31,57,122 \end{pmatrix},$$

$\max\limits_{i \in \sigma'} d_i = \begin{pmatrix} 10,20,30,40 \\ 9,20,30,41 \\ 8,20,30,42 \\ 7,20,30,43 \end{pmatrix}$. Since, this maximum value is not greater than

ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
(ISRA), Impact Factor: 2.114

$q_\sigma = \begin{pmatrix} 10,20,30,40 \\ 9,20,30,41 \\ 8,20,30,42 \\ 7,20,30,43 \end{pmatrix}$. The details of computations of the lower bound for each of the node is

$P_{i\sigma}^1$	$V_{i\sigma} = V_\sigma + \max(0, q_\sigma - d_i)$	$b_{i\sigma} = v_{i\sigma}$
P_{13}^2	$\begin{pmatrix} -21,14,48,83 \\ -32,14,48,94 \\ -43,14,48,105 \\ -54,14,48,116 \end{pmatrix} + \max\left[0, \begin{pmatrix} 2,31,57,86 \\ -10,31,57,98 \\ -22,31,57,110 \\ -34,31,57,122 \end{pmatrix} - \begin{pmatrix} 14,18,22,26 \\ 13,18,22,27 \\ 12,18,22,28 \\ 11,18,22,29 \end{pmatrix}\right]$	$\begin{pmatrix} -45,23,87,155 \\ -69,23,87,179 \\ -93,23,87,203 \\ -117,23,87,227 \end{pmatrix}$
P_{23}^2	$\begin{pmatrix} -21,14,48,83 \\ -32,14,48,94 \\ -43,14,48,105 \\ -54,14,48,116 \end{pmatrix} + \max\left[0, \begin{pmatrix} 2,31,57,86 \\ -10,31,57,98 \\ -22,31,57,110 \\ -34,31,57,122 \end{pmatrix} - \begin{pmatrix} 12,14,16,18 \\ 11,14,16,19 \\ 10,14,16,20 \\ 9,14,16,21 \end{pmatrix}\right]$	$\begin{pmatrix} -37,29,91,157 \\ -61,29,91,181 \\ -85,29,91,205 \\ -109,29,91,229 \end{pmatrix}$
P_{43}^2	$\begin{pmatrix} -21,14,48,83 \\ -32,14,48,94 \\ -43,14,48,105 \\ -54,14,48,116 \end{pmatrix} + \max\left[0, \begin{pmatrix} 2,31,57,86 \\ -10,31,57,98 \\ -22,31,57,110 \\ -34,31,57,122 \end{pmatrix} - \begin{pmatrix} 5,13,21,29 \\ 4,13,21,30 \\ 3,13,21,31 \\ 2,13,21,32 \end{pmatrix}\right]$	$\begin{pmatrix} -48,24,92,164 \\ -72,24,92,188 \\ -96,24,92,212 \\ -120,24,92,236 \end{pmatrix}$
P_{53}^2	$\begin{pmatrix} -21,14,48,83 \\ -32,14,48,94 \\ -43,14,48,105 \\ -54,14,48,116 \end{pmatrix} + \max\left[0, \begin{pmatrix} 2,31,57,86 \\ -10,31,57,98 \\ -22,31,57,110 \\ -34,31,57,122 \end{pmatrix} - \begin{pmatrix} 10,20,30,40 \\ 9,20,30,41 \\ 8,20,30,42 \\ 7,20,30,43 \end{pmatrix}\right]$	$\begin{pmatrix} -59,15,85,159 \\ -83,15,85,183 \\ -107,15,85,207 \\ -131,15,85,231 \end{pmatrix}$

Active list = { $P_5^1, P_1^1, P_4^1, P_2^1, P_{53}^2, P_{13}^2, P_{43}^2, P_{23}^2$ }.

Proceeding in this way, we get

Step- 7:
This subproblem occurs at level 3 which is not equal to {4}. Hence, check the dominance property, $\sigma = \{5,1,3\}$, $\sigma^1 =$ {2,4}, $q_{513} = q_{13} - t_5 = \begin{pmatrix} -16,15,43,74 \\ -29,15,43,87 \\ -42,15,43,100 \\ -55,15,43,113 \end{pmatrix} - \begin{pmatrix} 4,10,14,20 \\ 2,10,14,22 \\ 0,10,14,24 \\ -2,10,14,26 \end{pmatrix}$

ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
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= $\begin{pmatrix} -36,1,33,70 \\ -51,1,33,85 \\ -66,1,33,100 \\ -81,1,33,115 \end{pmatrix}$. $\max_{i \in \sigma'} d_i$= $\begin{pmatrix} -36,1,33,70 \\ -51,1,33,85 \\ -66,1,33,100 \\ -81,1,33,115 \end{pmatrix}$. Since, the maximum value is equal to

$q_\sigma \begin{pmatrix} -36,1,33,70 \\ -51,1,33,85 \\ -66,1,33,100 \\ -81,1,33,115 \end{pmatrix}$.

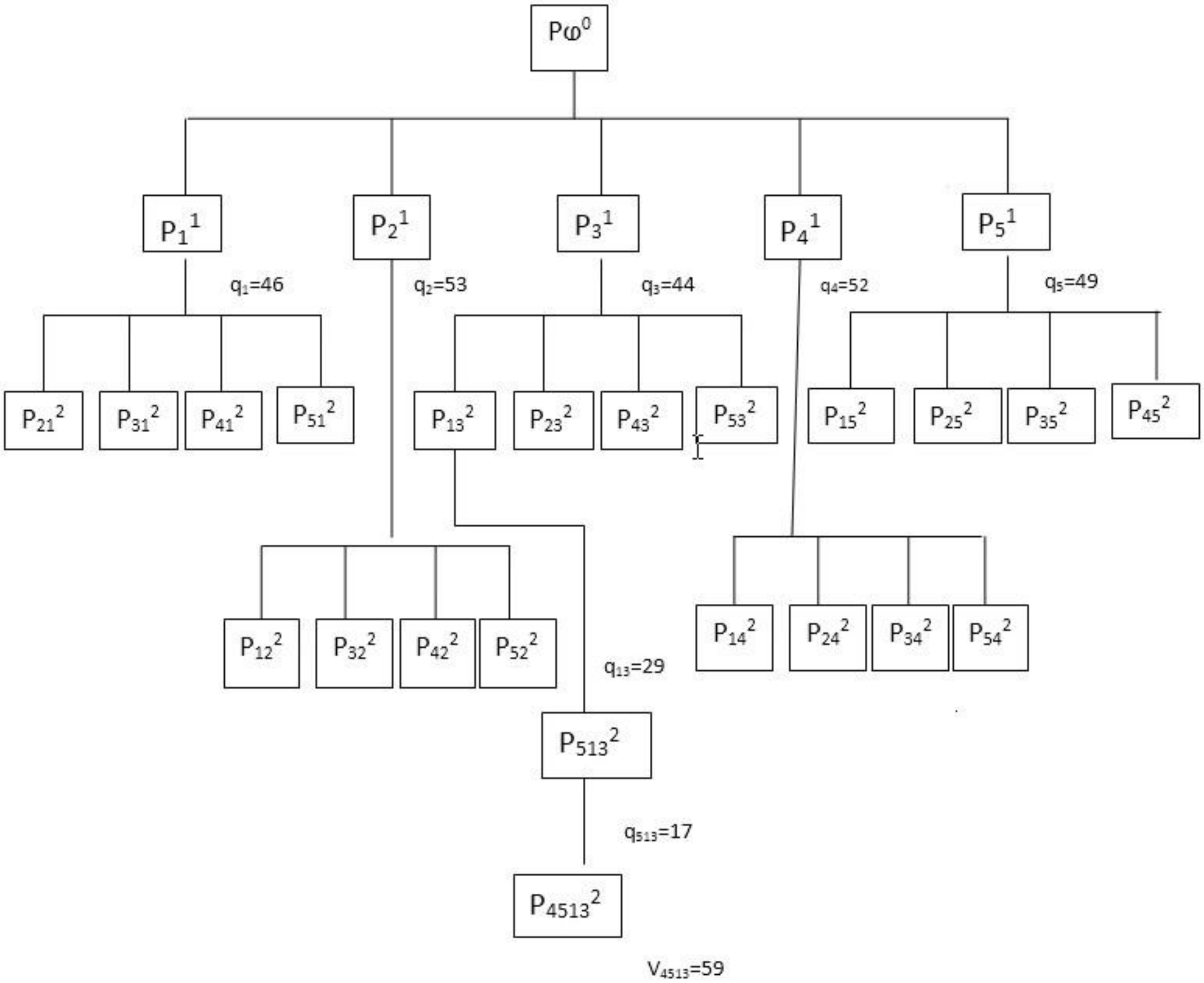
Step-8:

Job 4 has an element in σ^1 which has the highest due date. Hence based on the dominance property the subproblem P_{513}^3 is further partitioned with a single branch P_{4513}^4.σ= {5,1,3} &

j = 4, q_σ= $\begin{pmatrix} -36,1,33,70 \\ -51,1,33,85 \\ -66,1,33,100 \\ -81,1,33,115 \end{pmatrix}$, $V_{j\sigma} = V_\sigma + \max(0, q_\sigma - d_j)$

$$= \begin{pmatrix} -110,5,113,228 \\ -148,5,113,266 \\ -186,5,113,304 \\ -224,5,113,342 \end{pmatrix} + \max \left[0, \begin{pmatrix} -36,1,33,70 \\ -51,1,33,85 \\ -66,1,33,100 \\ -81,1,33,115 \end{pmatrix} - \begin{pmatrix} -36,1,33,70 \\ -51,1,33,85 \\ -66,1,33,100 \\ -81,1,33,115 \end{pmatrix} \right] = \begin{pmatrix} -110,5,113,228 \\ -148,5,113,266 \\ -186,5,113,304 \\ -224,5,113,342 \end{pmatrix}.$$

TREE DIAGRAM



The minimum total tardiness value is $\begin{pmatrix} -110,5,113,228 \\ -148,5,113,266 \\ -186,5,113,304 \\ -224,5,113,342 \end{pmatrix}$.

The minimum total tardiness value is 59.
The required optimal sequence is 2→4→5→1→3.

CONCLUSION

We considered single machine scheduling problem (SMSP) with fuzzy processing time and fuzzy due date to minimize the total tardiness. This method is very easy to understand each stage that will help the decision maker in determining a best schedule for a given set of jobs effectively. This method has significant use of practical results in industries.

REFERENCES

[1] Azizoglu M, Kondacki, Suna and omer., International Journal of Production Economics,23,1.1991.
[2] Emmons H, One machine sequencing to minimize certain functions of Tardiness, Operations Research, 17, 701-715, 1969.

ISSN: 2277-9655
Scientific Journal Impact Factor: 3.449
(ISRA), Impact Factor: 2.114

[3] Gawiejnowicz S. et.al., " Analysis of a time dependent scheduling problem by signatures of Deterioration rate sequences", Discrete Applied Mathematics, vol.154, 2150-2166,2006.
[4] Henri P. " Using fuzzy set theory in a scheduling problem: A case study. "Fuzzy sets and Systems", vol. 2, 153-165,1979.
[5] Hong T.P. and Chuang T.N. " A new triangular fuzzy Johnson algorithm," Computers and Application Industrial Engineering, vol.36, 179-200, 2000.
[6] Ishii H and Tada M., " Single machine scheduling problem with fuzzy precedence relation, " European Journal of Operational Research, vol.6, 141-147,2000.
[7] Itoh T. and Ishii H. " Fuzzy due-date scheduling problem with fuzzy processing time",International Transactions in Operational Research, vol.6, 639-647, 1999. .
[8] John R.J. "Type-2 fuzzy sets and approach of theory and applications", International Journal Fuzziness Knowledge Based Systems, 6(6), 563-576, 1998.
[9] Kuroda M and Wang Z, " Fuzzy job shop scheduling", International Journal of Production Economics, vol.44, 45-51,1996.
[10] Lawler, E.L, "A Pseudopolynomial algorithm for sequencing jobs to minimize total tardiness," Annals of Discrete Mathematics, 331-342, 1977.
[11] M.Mizumoto and K.Tanaka, " Some properties of fuzzy sets of type-2", Information and Control, vol.31,312-340,1976.
[12] J,M, Mendal F,Liu and D.Zhai, "α-plane representation for type-2 fuzzy sets: theory and applications", IEEE Trans. Fuzzy Systems, vol.17,1189-1207,2009.
[13] Q. Mendoza, P.Melin and G,Licea, " A hybrid approach for image recognition combining type-2 fuzzy logic, modular neural networks and the sugeno integral", Information Sciences, vol.179,2078-2101.
[14] Raymond B.," European Journal of Operational Research", 61(1-2), 213. 1992.
[15] Vairaklarakis G.L. and YEE, Lee Chung, IIE, Truncation, ,27(2), 1995.
[16] Zadeh I.A., "The fuzzy concept of a Linguistic variable and its applications to approximate Reasoning', International Journal Inform Science, 8, 199-240, 1997.